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Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

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Chapter 9

Complex numbers

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317. 9709_s20_qp_31 Q: 10

(a) The complex number u is defined by $u = \dfrac{3i}{a + 2i}$, where a is real.

(i) Express u in the Cartesian form $x + iy$, where x and y are in terms of a. [3]

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(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

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(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leqslant |z - 1 - i|$ and $|z - 2 - i| \leqslant 2$. [4]

(ii) Calculate the least value of $\arg z$ for points in this region. [2]

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318. 9709_s20_qp_32 Q: 8

(a) Solve the equation $(1+2\mathrm{i})w + \mathrm{i}w^* = 3 + 5\mathrm{i}$. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [4]

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(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leqslant 1$ and $\arg(z - 4i) \geqslant -\frac{1}{4}\pi$. [4]

(ii) Find the least value of $\operatorname{Im} z$ for points in this region, giving your answer in an exact form. [2]

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319. 9709_s20_qp_33 Q: 9

(a) The complex numbers u and w are such that

$$u - w = 2\mathrm{i} \quad \text{and} \quad uw = 6.$$

Find u and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

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(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2i| \leqslant 2, \quad 0 \leqslant \arg z \leqslant \tfrac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leqslant 3.$$ [5]

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320. 9709_w20_qp_31 Q: 2

On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geqslant 2$ and $|z - 1 + i| \leqslant 1$. [4]

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321. 9709_w20_qp_31 Q: 7

(a) Verify that $-1 + \sqrt{5}\mathrm{i}$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

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(b) Find the other roots of this equation. [4]

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322. 9709_w20_qp_32 Q: 6

The complex number u is defined by

$$u = \frac{7 + \mathrm{i}}{1 - \mathrm{i}}.$$

(a) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7 + \mathrm{i}$ and $1 - \mathrm{i}$ respectively. [2]

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(c) By considering the arguments of 7 + i and 1 – i, show that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = \tan^{-1}\left(\tfrac{1}{7}\right) + \tfrac{1}{4}\pi.$$ [3]

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323. 9709_m19_qp_32 Q: 7

(a) Showing all working and without using a calculator, solve the equation

$$(1 + \mathrm{i})z^2 - (4 + 3\mathrm{i})z + 5 + \mathrm{i} = 0.$$

Give your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

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(b) The complex number u is given by

$$u = -1 - \mathrm{i}.$$

On a sketch of an Argand diagram show the point representing u. Shade the region whose points represent complex numbers satisfying the inequalities $|z| < |z - 2\mathrm{i}|$ and $\frac{1}{4}\pi < \arg(z - u) < \frac{1}{2}\pi$. [4]

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324. 9709_s19_qp_31 Q: 10

Throughout this question the use of a calculator is not permitted.

The complex number $(\sqrt{3}) + \mathrm{i}$ is denoted by u.

(i) Express u in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. Hence or otherwise state the exact values of the modulus and argument of u^4. [5]

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(ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant 2$ and $\text{Im}\, z \geqslant 2$, where $\text{Im}\, z$ denotes the imaginary part of z. [5]

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325. 9709_s19_qp_32 Q: 5

Throughout this question the use of a calculator is not permitted.

It is given that the complex number $-1 + (\sqrt{3})\mathrm{i}$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

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(ii) Find the value of k and the third root of the equation. [6]

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326. 9709_s19_qp_33 Q: 8

Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{4i}{1 - (\sqrt{3})i}.$$

(i) Express u in the form $x + iy$, where x and y are real and exact. [3]

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(ii) Find the exact modulus and argument of u. [2]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

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327. 9709_w19_qp_31 Q: 10

(a) The complex number u is given by $u = -3 - (2\sqrt{10})i$. Showing all necessary working and without using a calculator, find the square roots of u. Give your answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

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(b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - \mathrm{i}| \leqslant 3$, $\arg z \geqslant \frac{1}{4}\pi$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of the complex number z. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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328. 9709_w19_qp_32 Q: 7

(a) Find the complex number z satisfying the equation

$$z + \frac{iz}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z. Give your answer in the form $x + iy$, where x and y are real. [5]

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(b) **(i)** On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im}\, z = 3$, where $\text{Im}\, z$ denotes the imaginary part of z. [2]

(ii) In the first quadrant the two loci intersect at the point P. Find the exact argument of the complex number represented by P. [2]

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329. 9709_w19_qp_33 Q: 6

Throughout this question the use of a calculator is not permitted.

The complex number with modulus 1 and argument $\frac{1}{3}\pi$ is denoted by w.

(i) Express w in the form $x + iy$, where x and y are real and exact. [1]

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The complex number $1 + 2i$ is denoted by u. The complex number v is such that $|v| = 2|u|$ and $\arg v = \arg u + \frac{1}{3}\pi$.

(ii) Sketch an Argand diagram showing the points representing u and v. [2]

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(iii) Explain why v can be expressed as $2uw$. Hence find v, giving your answer in the form $a + \mathrm{i}b$, where a and b are real and exact. [4]

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330. 9709_m18_qp_32 Q: 9

The complex number $1 + 2i$ is denoted by u.

(i) It is given that u is a root of the equation $2x^3 - x^2 + 4x + k = 0$, where k is a constant.

(a) Showing all working and without using a calculator, find the value of k. [3]

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(b) Showing all working and without using a calculator, find the other two roots of this equation. [4]

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(ii) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 1$. Determine the least value of $\arg z$ for points on this locus. Give your answer in radians correct to 2 decimal places. [4]

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331. 9709_s18_qp_31 Q: 7

(i) Showing all working and without using a calculator, solve the equation $z^2 + (2\sqrt{6})z + 8 = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [3]

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(ii) Sketch an Argand diagram showing the points representing the roots. [1]

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(iii) The points representing the roots are A and B, and O is the origin. Find angle AOB. [3]

(iv) Prove that triangle AOB is equilateral. [1]

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332. 9709_s18_qp_32 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex numbers $-3\sqrt{3} + \mathrm{i}$ and $\sqrt{3} + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real and exact, the complex numbers uv and $\frac{u}{v}$. [5]

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(ii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]

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333. 9709_s18_qp_33 Q: 9

(a) Find the complex number z satisfying the equation

$$3z - iz^* = 1 + 5i,$$

where z^* denotes the complex conjugate of z. [4]

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(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| \leqslant 3$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of z. Calculate the greatest value of $\arg z$ for points in this region. Give your answer in radians correct to 2 decimal places. [5]

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334. 9709_w18_qp_31 Q: 8

(a) Showing all necessary working, express the complex number $\frac{2+3\text{i}}{1-2\text{i}}$ in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the values of r and θ correct to 3 significant figures. [5]

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(b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2i| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

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335. 9709_w18_qp_32 Q: 9

(a) **(i)** Without using a calculator, express the complex number $\frac{2+6i}{1-2i}$ in the form $x + iy$, where x and y are real. [2]

(ii) Hence, without using a calculator, express $\frac{2+6i}{1-2i}$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

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(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - 3i| \leqslant 1$ and $\text{Re}\, z \leqslant 0$, where $\text{Re}\, z$ denotes the real part of z. Find the greatest value of $\arg z$ for points in this region, giving your answer in radians correct to 2 decimal places. [5]

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336. 9709_m17_qp_32 Q: 8

Throughout this question the use of a calculator is not permitted.

The polynomial $z^4 + 3z^2 + 6z + 10$ is denoted by $\text{p}(z)$. The complex number $-1 + \text{i}$ is denoted by u.

(i) Showing all your working, verify that u is a root of the equation $\text{p}(z) = 0$. [3]

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(ii) Find the other three roots of the equation $\text{p}(z) = 0$. [7]

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337. 9709_s17_qp_31 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex numbers u and w are defined by $u = -1 + 7\mathrm{i}$ and $w = 3 + 4\mathrm{i}$.

(i) Showing all your working, find in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers $u - 2w$ and $\frac{u}{w}$. [4]

In an Argand diagram with origin O, the points A, B and C represent the complex numbers u, w and $u - 2w$ respectively.

(ii) Prove that angle $AOB = \frac{1}{4}\pi$. [2]

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(iii) State fully the geometrical relation between the line segments OB and CA. [2]

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338. 9709_s17_qp_32 Q: 6

Throughout this question the use of a calculator is not permitted.

The complex number 2 – i is denoted by u.

(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

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(ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + \mathrm{i}|$. [4]

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339. 9709_s17_qp_33 Q: 11

Throughout this question the use of a calculator is not permitted.

(a) The complex numbers z and w satisfy the equations

$$z + (1 + \mathrm{i})w = \mathrm{i} \qquad \text{and} \qquad (1 - \mathrm{i})z + \mathrm{i}w = 1.$$

Solve the equations for z and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

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(b) The complex numbers u and v are given by $u = 1 + (2\sqrt{3})\mathrm{i}$ and $v = 3 + 2\mathrm{i}$. In an Argand diagram, u and v are represented by the points A and B. A third point C lies in the first quadrant and is such that $BC = 2AB$ and angle $ABC = 90°$. Find the complex number z represented by C, giving your answer in the form $x + \mathrm{i}y$, where x and y are real and exact. [4]

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340. 9709_w17_qp_31 Q: 7

(a) The complex number u is given by $u = 8 - 15\text{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \text{i}b$, where the numbers a and b are real and exact. [5]

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(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

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341. 9709_w17_qp_32 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex number $1 - (\sqrt{3})i$ is denoted by u.

(i) Find the modulus and argument of u. [2]

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(ii) Show that $u^3 + 8 = 0$. [2]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leqslant 2$ and $\mathrm{Re}\, z \geqslant 2$, where $\mathrm{Re}\, z$ denotes the real part of z. [4]

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342. 9709_m16_qp_32 Q: 10

(a) Find the complex number z satisfying the equation $z^* + 1 = 2\mathrm{i}z$, where z^* denotes the complex conjugate of z. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z + 1 - 3\mathrm{i}| \leqslant 1$ and $\mathrm{Im}\, z \geqslant 3$, where $\mathrm{Im}\, z$ denotes the imaginary part of z. [4]

(ii) Determine the difference between the greatest and least values of $\arg z$ for points lying in this region. [2]

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343. 9709_s16_qp_31 Q: 10

(a) Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

(b) **(i)** On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2\mathrm{i}| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]

(ii) Calculate the least value of $|w - z|$ for points on these loci. [2]

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344. 9709_s16_qp_32 Q: 10

(a) Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) **(i)** On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

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345. 9709_s16_qp_33 Q: 9

Throughout this question the use of a calculator is not permitted.

The complex numbers $-1 + 3i$ and $2 - i$ are denoted by u and v respectively. In an Argand diagram with origin O, the points A, B and C represent the numbers u, v and $u + v$ respectively.

(i) Sketch this diagram and state fully the geometrical relationship between OB and AC. [4]

(ii) Find, in the form $x + iy$, where x and y are real, the complex number $\frac{u}{v}$. [3]

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

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346. 9709_w16_qp_31 Q: 9

Throughout this question the use of a calculator is not permitted.

(a) Solve the equation $(1 + 2\mathrm{i})w^2 + 4w - (1 - 2\mathrm{i}) = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - \mathrm{i}| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

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347. 9709_w16_qp_33 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex number z is defined by $z = (\sqrt{2}) - (\sqrt{6})\text{i}$. The complex conjugate of z is denoted by z^*.

(i) Find the modulus and argument of z. [2]

(ii) Express each of the following in the form $x + \text{i}y$, where x and y are real and exact:

(a) $z + 2z^*$;

(b) $\dfrac{z^*}{\text{i}z}$.

[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z^* and $\text{i}z$ respectively. Prove that angle AOB is equal to $\frac{1}{6}\pi$. [3]

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348. 9709_s15_qp_31 Q: 8

The complex number w is defined by $w = \dfrac{22 + 4\mathrm{i}}{(2 - \mathrm{i})^2}$.

(i) Without using a calculator, show that $w = 2 + 4\mathrm{i}$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leqslant \arg(w + p) \leqslant \frac{3}{4}\pi$. Find the set of possible values of p. [3]

(iii) The complex conjugate of w is denoted by w^*. The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S, T and the origin. [3]

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349. 9709_s15_qp_32 Q: 7

The complex number u is given by $u = -1 + (4\sqrt{3})\text{i}$.

(i) Without using a calculator and showing all your working, find the two square roots of u. Give your answers in the form $a + \text{i}b$, where the real numbers a and b are exact. [5]

(ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]

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350. 9709_s15_qp_33 Q: 8

The complex number $1 - i$ is denoted by u.

(i) Showing your working and without using a calculator, express

$$\frac{i}{u}$$

in the form $x + iy$, where x and y are real. [2]

(ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - i| = 2$. [4]

(iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part **(ii)**. [3]

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351. 9709_w15_qp_31 Q: 9

The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\frac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

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352. 9709_w15_qp_33 Q: 9

(a) It is given that $(1 + 3i)w = 2 + 4i$. Showing all necessary working, prove that the exact value of $|w^2|$ is 2 and find $\arg(w^2)$ correct to 3 significant figures. [6]

(b) On a single Argand diagram sketch the loci $|z| = 5$ and $|z - 5| = |z|$. Hence determine the complex numbers represented by points common to both loci, giving each answer in the form $re^{i\theta}$. [4]

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