

Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS



Chapter 9

Complex numbers



317. 9709_s20_qp_31 Q: 10

(a) The complex number u is defined by $u = \frac{3i}{a+2i}$, where a is real.

(i) Express u in the Cartesian form $x + iy$, where x and y are in terms of a . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leq |z - 1 - i|$ and $|z - 2 - i| \leq 2$. [4]

- (ii) Calculate the least value of $\arg z$ for points in this region. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leq 1$ and $\arg(z - 4i) \geq -\frac{1}{4}\pi$. [4]

- (ii) Find the least value of $\text{Im } z$ for points in this region, giving your answer in an exact form. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

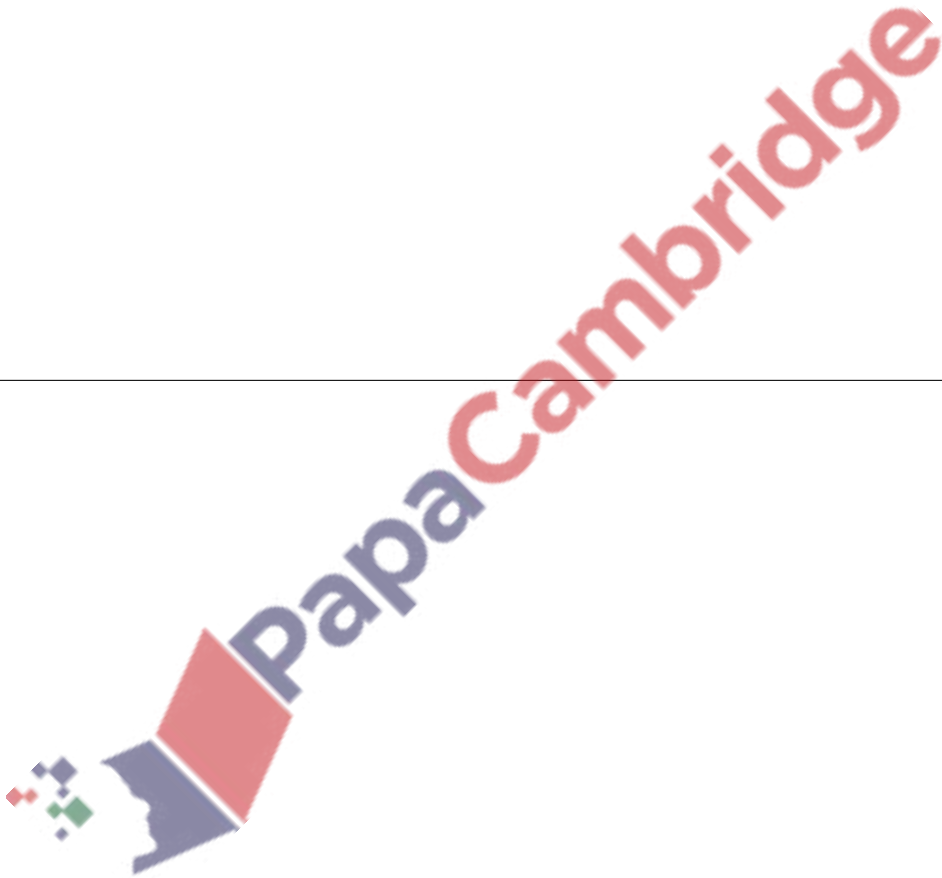
.....

- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2i| \leq 2, \quad 0 \leq \arg z \leq \frac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leq 3. \quad [5]$$

320. 9709_w20_qp_31 Q: 2

- On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. [4]



322. 9709_w20_qp_32 Q: 6

The complex number u is defined by

$$u = \frac{7+i}{1-i}.$$

- (a) Express u in the form $x + iy$, where x and y are real. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

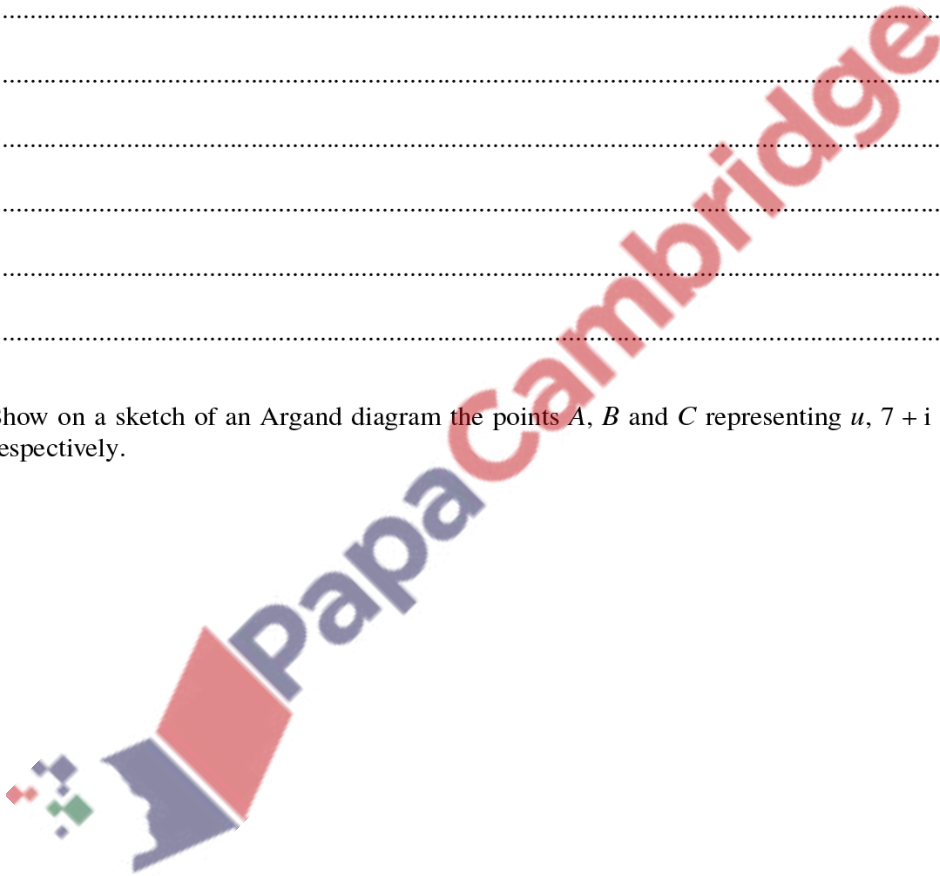
.....

.....

.....

.....

- (b) Show on a sketch of an Argand diagram the points A , B and C representing u , $7 + i$ and $1 - i$ respectively. [2]



323. 9709_m19_qp_32 Q: 7

- (a) Showing all working and without using a calculator, solve the equation

$$(1 + i)z^2 - (4 + 3i)z + 5 + i = 0.$$

Give your answers in the form $x + iy$, where x and y are real. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

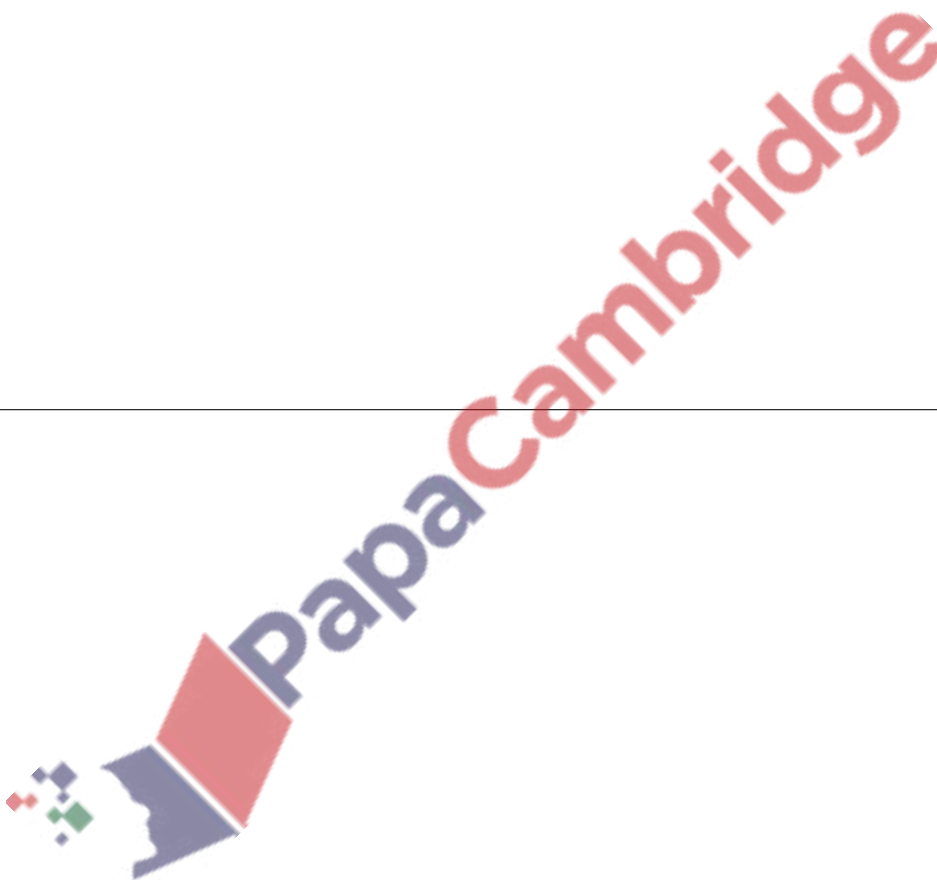
.....

.....

(b) The complex number u is given by

$$u = -1 - i.$$

On a sketch of an Argand diagram show the point representing u . Shade the region whose points represent complex numbers satisfying the inequalities $|z| < |z - 2i|$ and $\frac{1}{4}\pi < \arg(z - u) < \frac{1}{2}\pi$. [4]



324. 9709_s19_qp_31 Q: 10

Throughout this question the use of a calculator is not permitted.

The complex number $(\sqrt{3}) + i$ is denoted by u .

- (i) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . Hence or otherwise state the exact values of the modulus and argument of u^4 . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

.....

.....

.....

.....

.....

.....

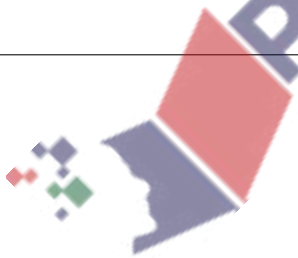
.....

.....

.....

.....

- (iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leq 2$ and $\text{Im } z \geq 2$, where $\text{Im } z$ denotes the imaginary part of z . [5]



325. 9709_s19_qp_32 Q: 5

Throughout this question the use of a calculator is not permitted.

It is given that the complex number $-1 + (\sqrt{3})i$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

- (i) Write down another root of the equation. [1]

.....

.....

.....

.....

.....

- (ii) Find the value of k and the third root of the equation. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the exact modulus and argument of u .

[2]

.....

.....

.....

.....

.....

.....

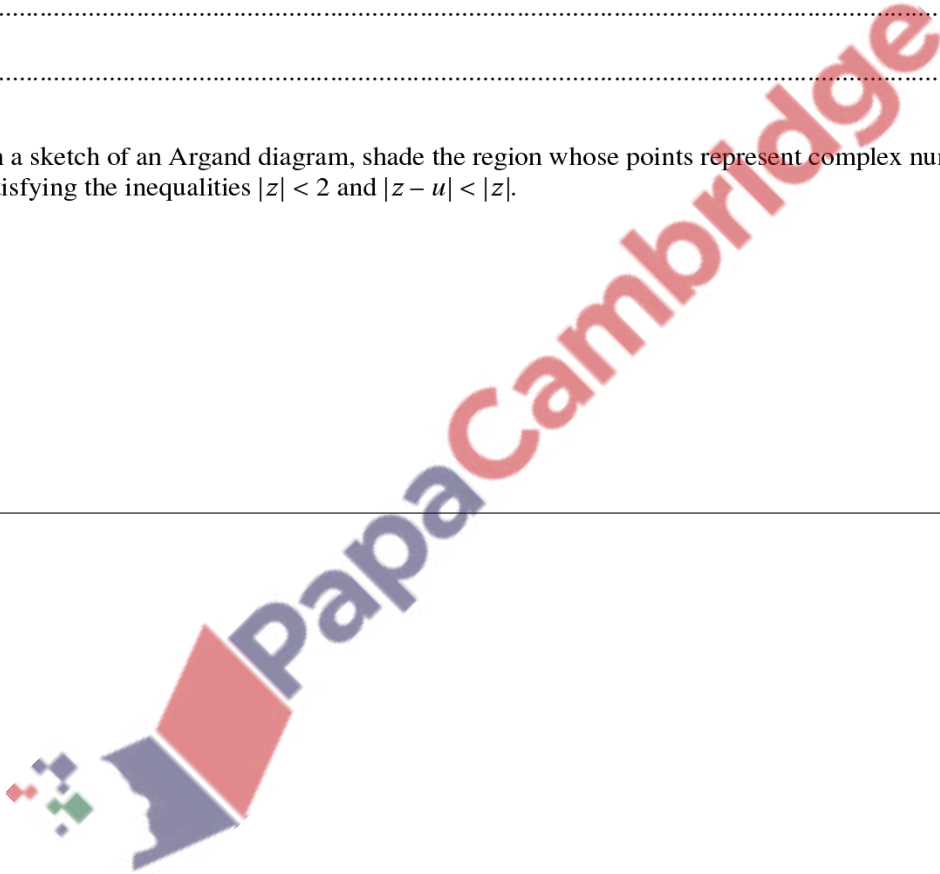
.....

.....

.....

.....

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]



327. 9709_w19_qp_31 Q: 10

- (a) The complex number u is given by $u = -3 - (2\sqrt{10})i$. Showing all necessary working and without using a calculator, find the square roots of u . Give your answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$, $\arg z \geq \frac{1}{4}\pi$ and $\text{Im } z \geq 2$, where $\text{Im } z$ denotes the imaginary part of the complex number z . [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

328. 9709_w19_qp_32 Q: 7

(a) Find the complex number z satisfying the equation

$$z + \frac{iz}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z . Give your answer in the form $x + iy$, where x and y are real. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) (i) On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im } z = 3$, where $\text{Im } z$ denotes the imaginary part of z . [2]

- (ii) In the first quadrant the two loci intersect at the point P . Find the exact argument of the complex number represented by P . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

329. 9709_w19_qp_33 Q: 6

Throughout this question the use of a calculator is not permitted.

The complex number with modulus 1 and argument $\frac{1}{3}\pi$ is denoted by w .

- (i) Express w in the form $x + iy$, where x and y are real and exact. [1]

.....

.....

.....

.....

.....

.....

.....

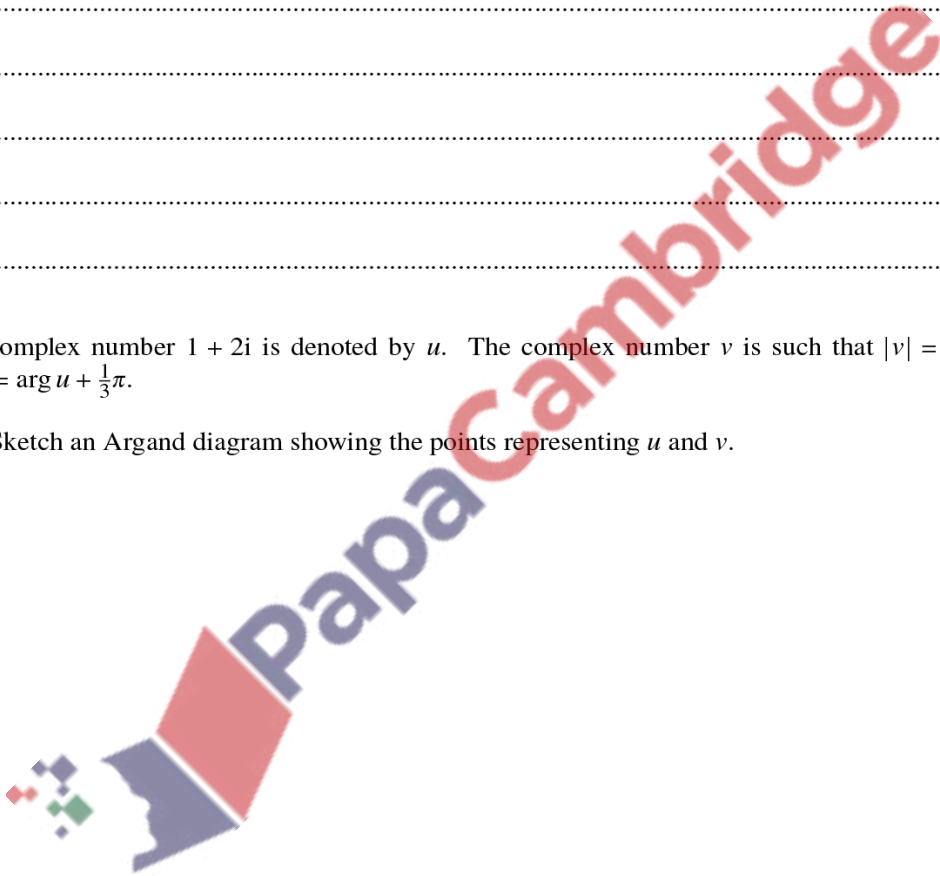
.....

.....

.....

The complex number $1 + 2i$ is denoted by u . The complex number v is such that $|v| = 2|u|$ and $\arg v = \arg u + \frac{1}{3}\pi$.

- (ii) Sketch an Argand diagram showing the points representing u and v . [2]



- (iii) Explain why v can be expressed as $2uw$. Hence find v , giving your answer in the form $a + ib$, where a and b are real and exact. [4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

PapaCambridge

330. 9709_m18_qp_32 Q: 9

The complex number $1 + 2i$ is denoted by u .(i) It is given that u is a root of the equation $2x^3 - x^2 + 4x + k = 0$, where k is a constant.(a) Showing all working and without using a calculator, find the value of k . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Showing all working and without using a calculator, find the other two roots of this equation. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 1$. Determine the least value of $\arg z$ for points on this locus. Give your answer in radians correct to 2 decimal places. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) The points representing the roots are A and B , and O is the origin. Find angle AOB . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iv) Prove that triangle AOB is equilateral. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

332. 9709_s18_qp_32 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex numbers $-3\sqrt{3} + i$ and $\sqrt{3} + 2i$ are denoted by u and v respectively.

- (i) Find, in the form $x + iy$, where x and y are real and exact, the complex numbers uv and $\frac{u}{v}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

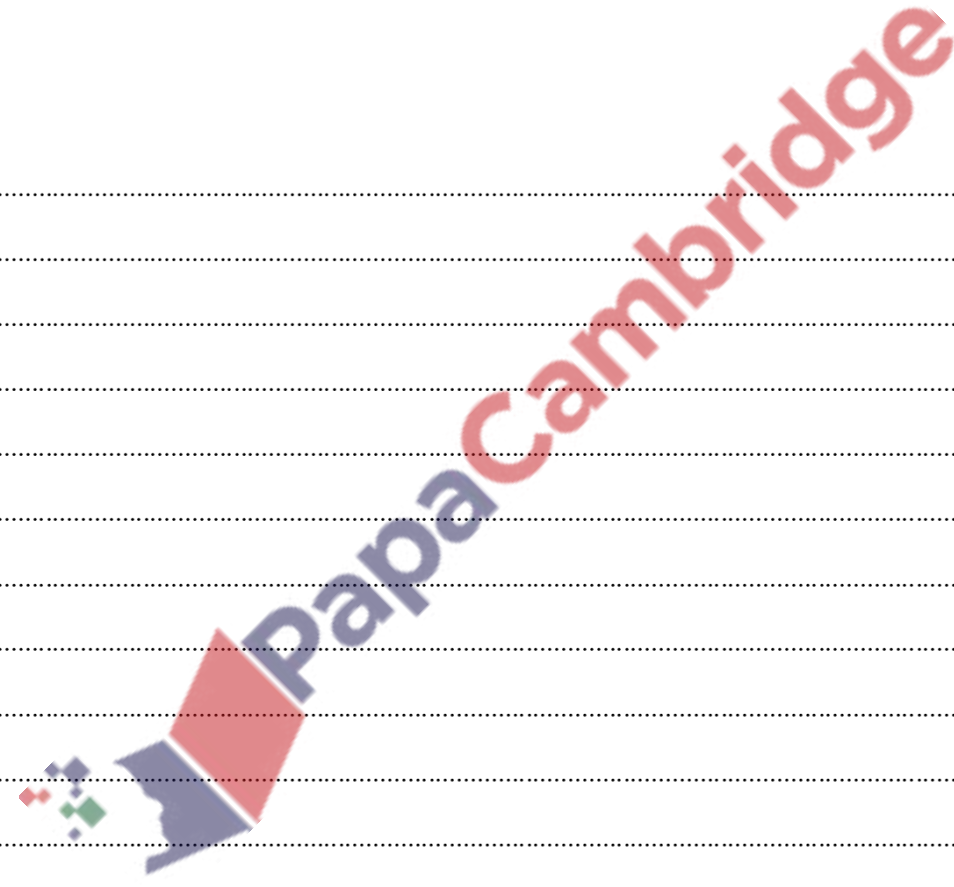
.....

.....

.....

.....

- (ii) On a sketch of an Argand diagram with origin O , show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]



A series of horizontal dotted lines for writing the solution to the problem.

333. 9709_s18_qp_33 Q: 9

- (a) Find the complex number z satisfying the equation

$$3z - iz^* = 1 + 5i,$$

where z^* denotes the complex conjugate of z .

[4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| \leq 3$ and $\text{Im } z \geq 2$, where $\text{Im } z$ denotes the imaginary part of z . Calculate the greatest value of $\arg z$ for points in this region. Give your answer in radians correct to 2 decimal places. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

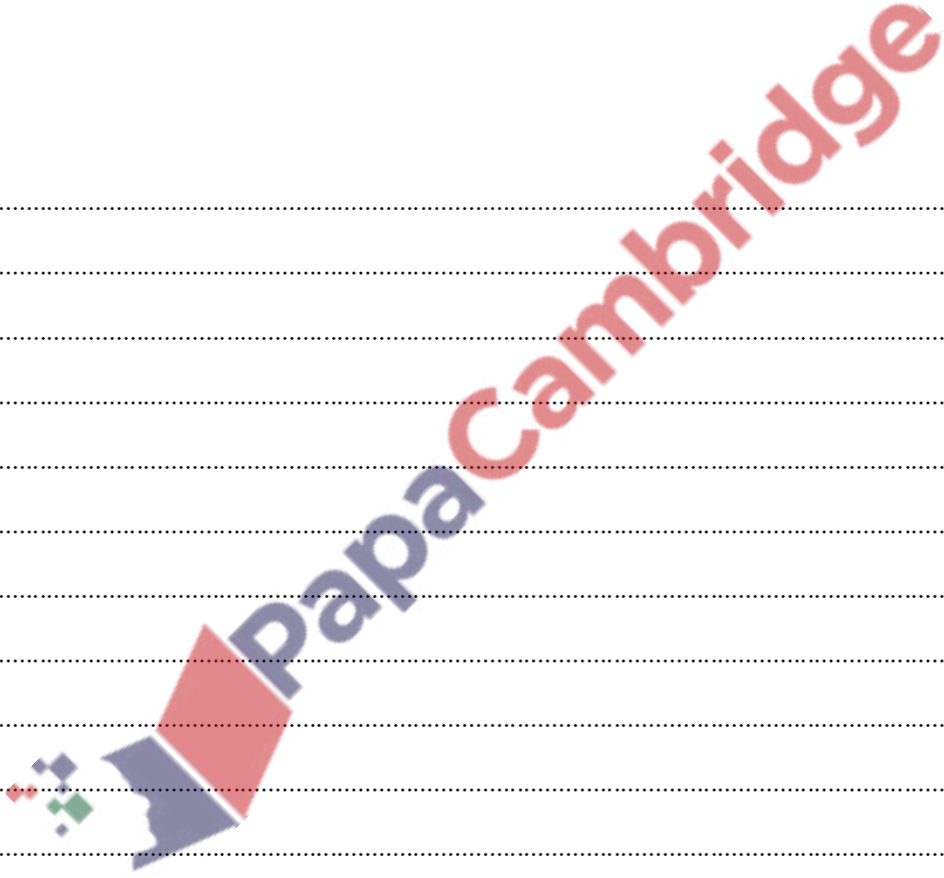
.....

.....

.....

.....

- (b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2i| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]



A series of horizontal dotted lines for writing the answer.

335. 9709_w18_qp_32 Q: 9

- (a) (i) Without using a calculator, express the complex number $\frac{2+6i}{1-2i}$ in the form $x+iy$, where x and y are real. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Hence, without using a calculator, express $\frac{2+6i}{1-2i}$ in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

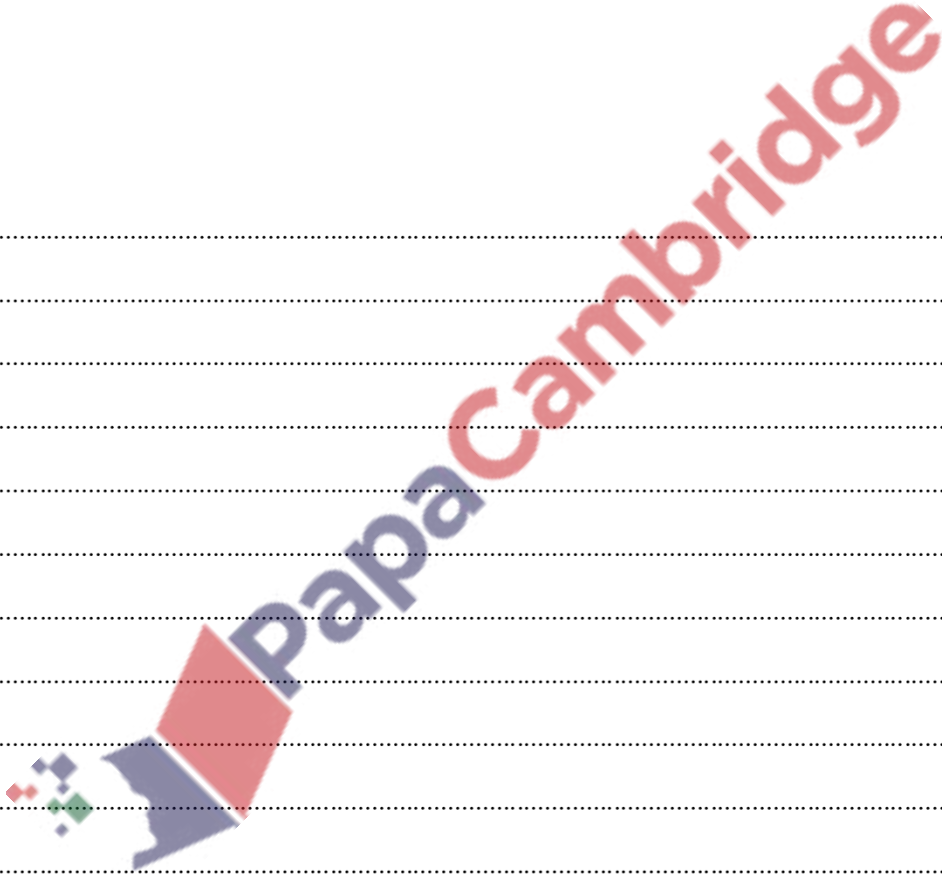
.....

.....

.....

.....

- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - 3i| \leq 1$ and $\operatorname{Re} z \leq 0$, where $\operatorname{Re} z$ denotes the real part of z . Find the greatest value of $\arg z$ for points in this region, giving your answer in radians correct to 2 decimal places. [5]



A series of horizontal dotted lines for writing the answer.

336. 9709_m17_qp_32 Q: 8

Throughout this question the use of a calculator is not permitted.

The polynomial $z^4 + 3z^2 + 6z + 10$ is denoted by $p(z)$. The complex number $-1 + i$ is denoted by u .

- (i) Showing all your working, verify that u is a root of the equation $p(z) = 0$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Find the other three roots of the equation $p(z) = 0$. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

337. 9709_s17_qp_31 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex numbers u and w are defined by $u = -1 + 7i$ and $w = 3 + 4i$.

- (i) Showing all your working, find in the form $x + iy$, where x and y are real, the complex numbers $u - 2w$ and $\frac{u}{w}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

In an Argand diagram with origin O , the points A , B and C represent the complex numbers u , w and $u - 2w$ respectively.

- (ii) Prove that angle $AOB = \frac{1}{4}\pi$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

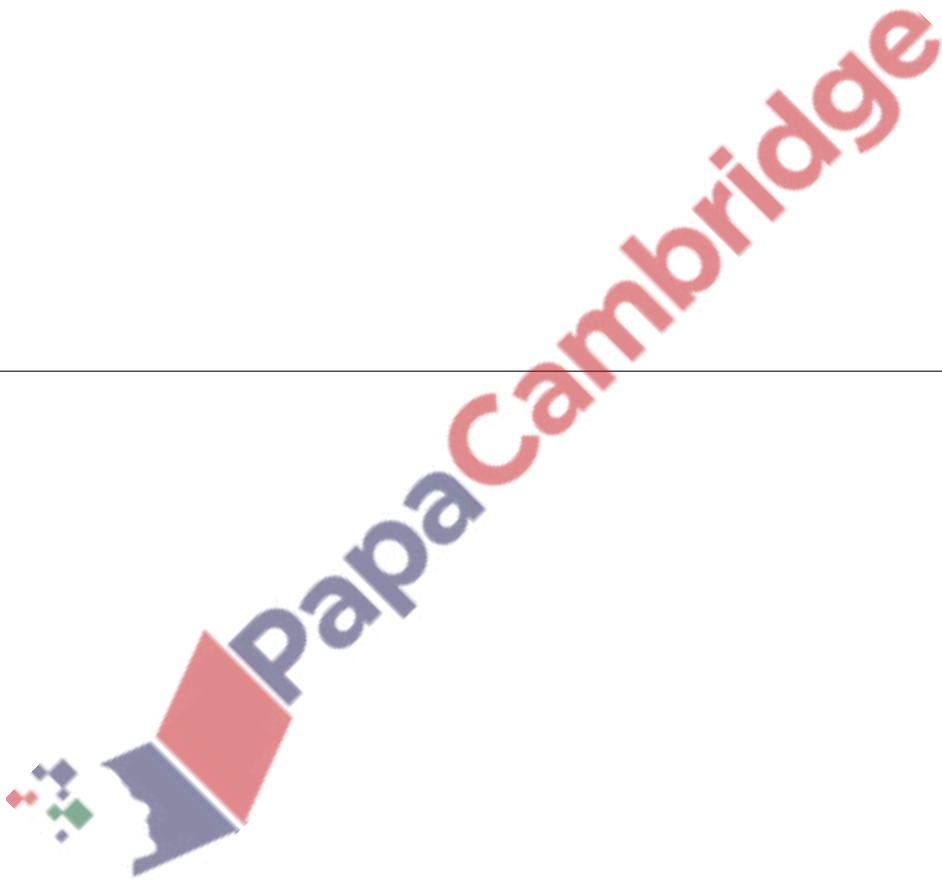
.....

.....

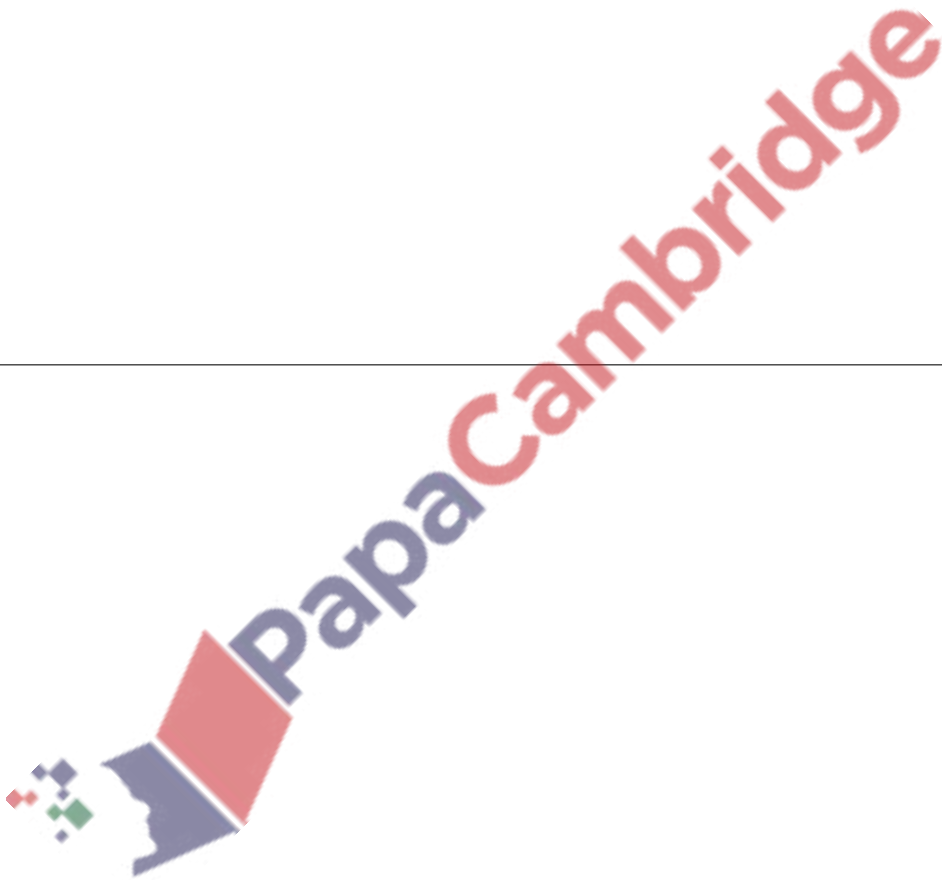
.....

.....

- (ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + i|$. [4]



- (b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - i| \leq 2$ and $0 \leq \arg(z - i) \leq \frac{1}{4}\pi$. [4]



341. 9709_w17_qp_32 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex number $1 - (\sqrt{3})i$ is denoted by u .

- (i) Find the modulus and argument of u . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Show that $u^3 + 8 = 0$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

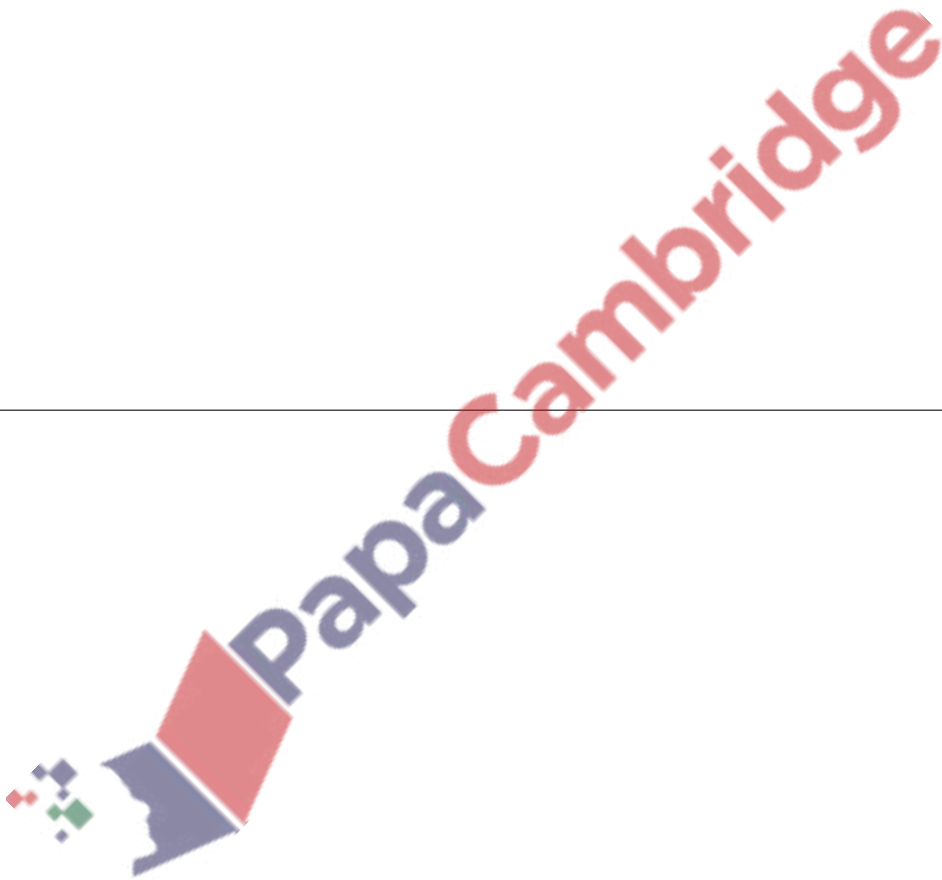
.....

.....

.....

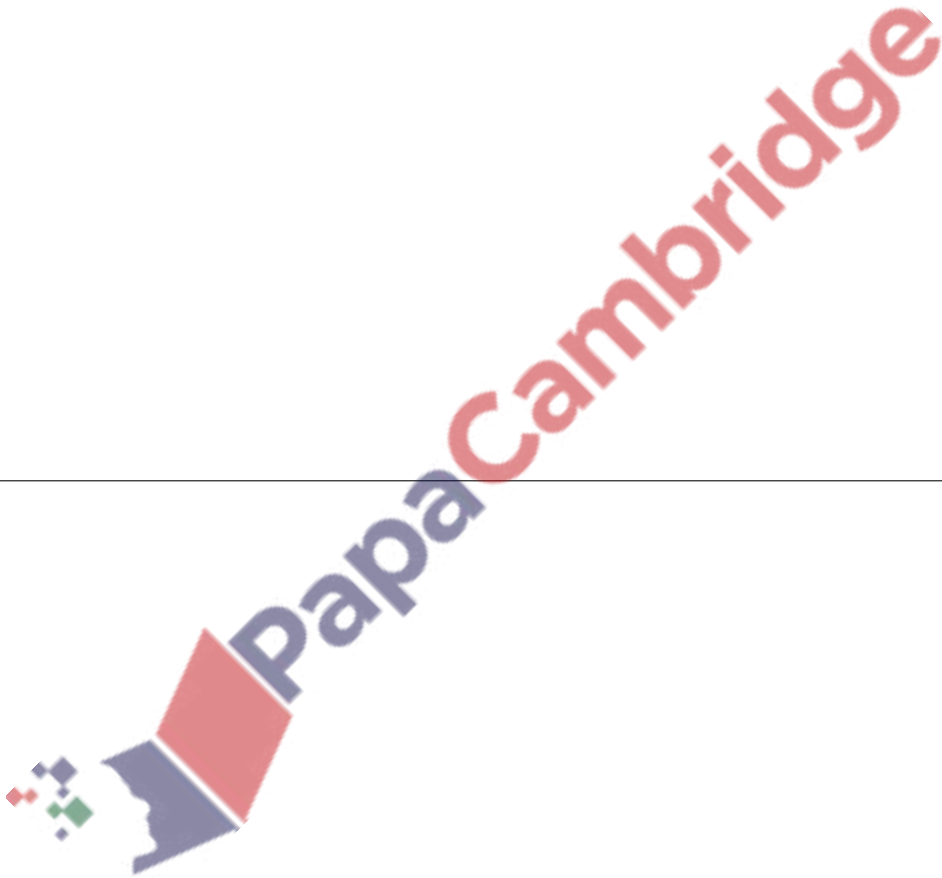
- (iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leq 2$ and $\operatorname{Re} z \geq 2$, where $\operatorname{Re} z$ denotes the real part of z .

[4]



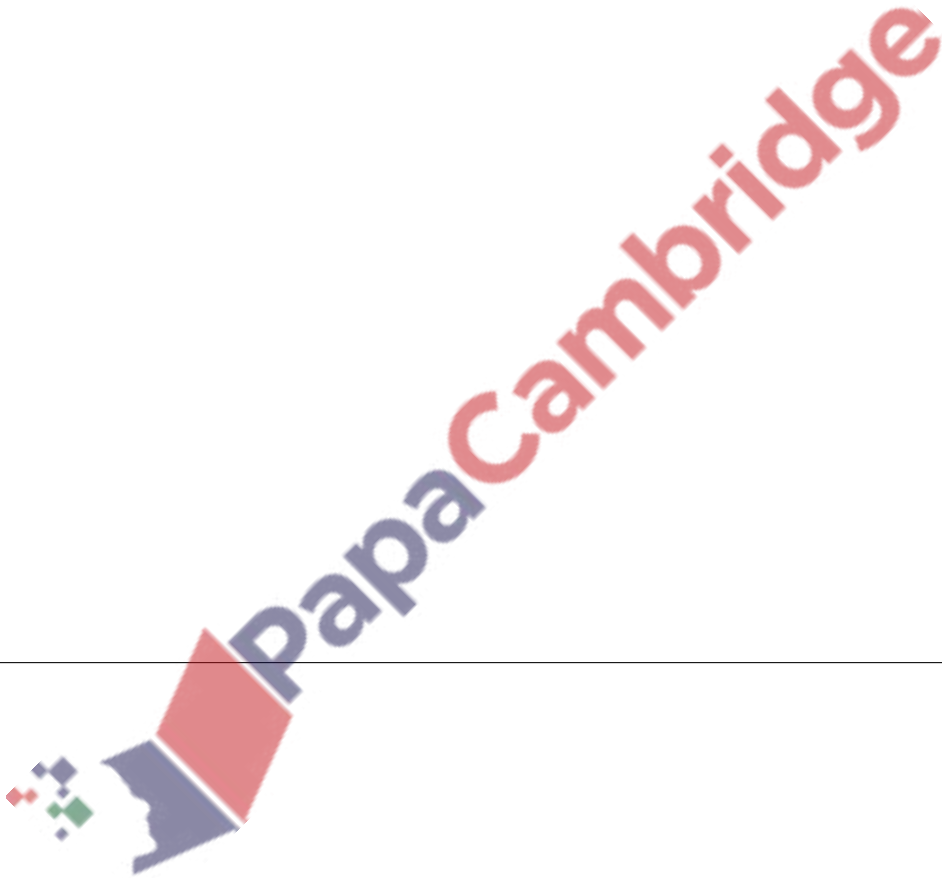
342. 9709_m16_qp_32 Q: 10

- (a) Find the complex number z satisfying the equation $z^* + 1 = 2iz$, where z^* denotes the complex conjugate of z . Give your answer in the form $x + iy$, where x and y are real. [5]
- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z + 1 - 3i| \leq 1$ and $\text{Im } z \geq 3$, where $\text{Im } z$ denotes the imaginary part of z . [4]
- (ii) Determine the difference between the greatest and least values of $\arg z$ for points lying in this region. [2]



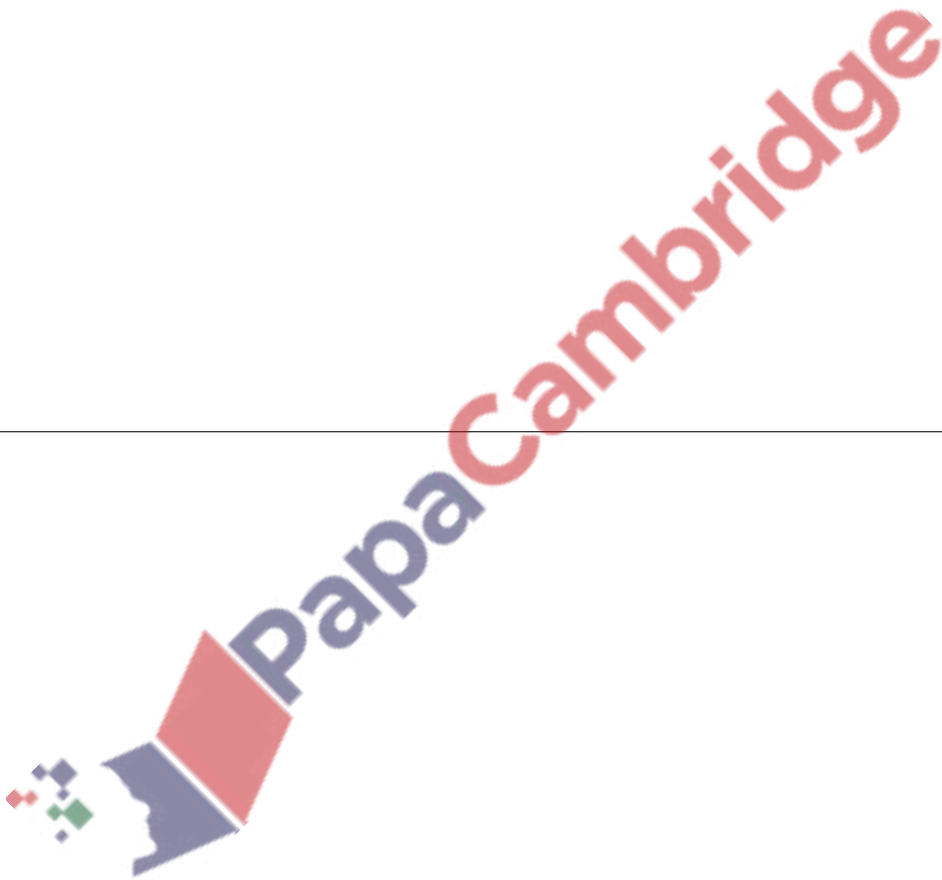
343. 9709_s16_qp_31 Q: 10

- (a) Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})i$. Give your answers in the form $x + iy$, where x and y are real and exact. [5]
- (b) (i) On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2i| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]
- (ii) Calculate the least value of $|w - z|$ for points on these loci. [2]



344. 9709_s16_qp_32 Q: 10

- (a) Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]
- (b) (i) On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]
- (ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

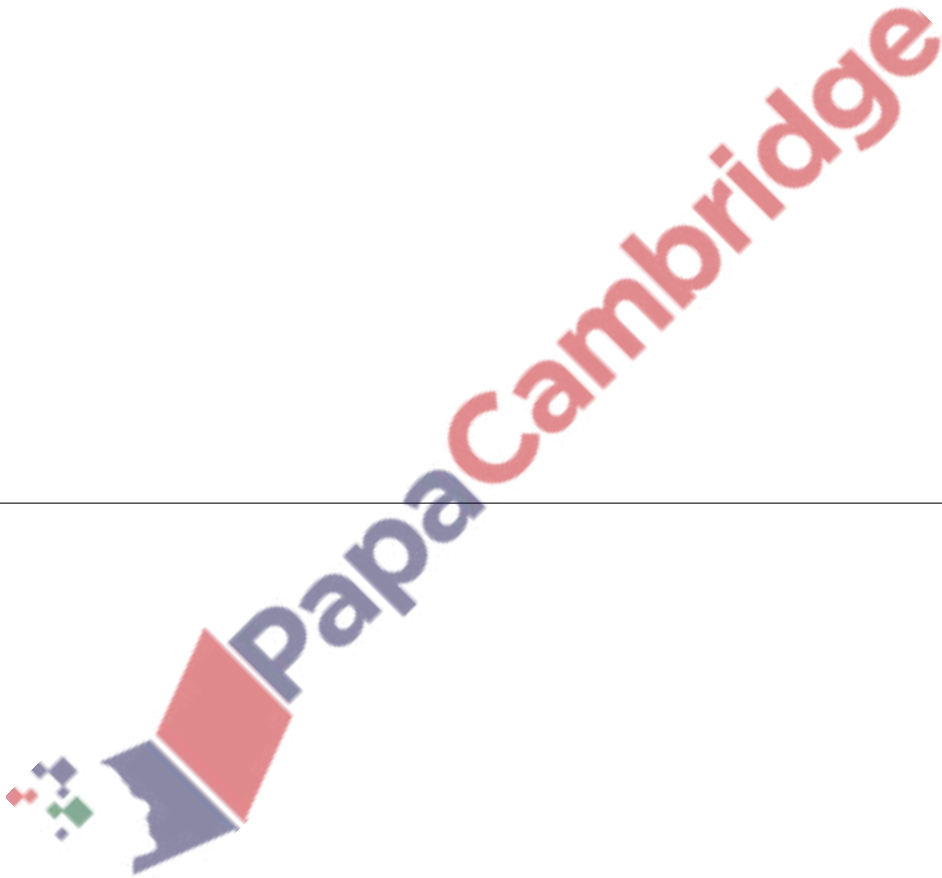


345. 9709_s16_qp_33 Q: 9

Throughout this question the use of a calculator is not permitted.

The complex numbers $-1 + 3i$ and $2 - i$ are denoted by u and v respectively. In an Argand diagram with origin O , the points A , B and C represent the numbers u , v and $u + v$ respectively.

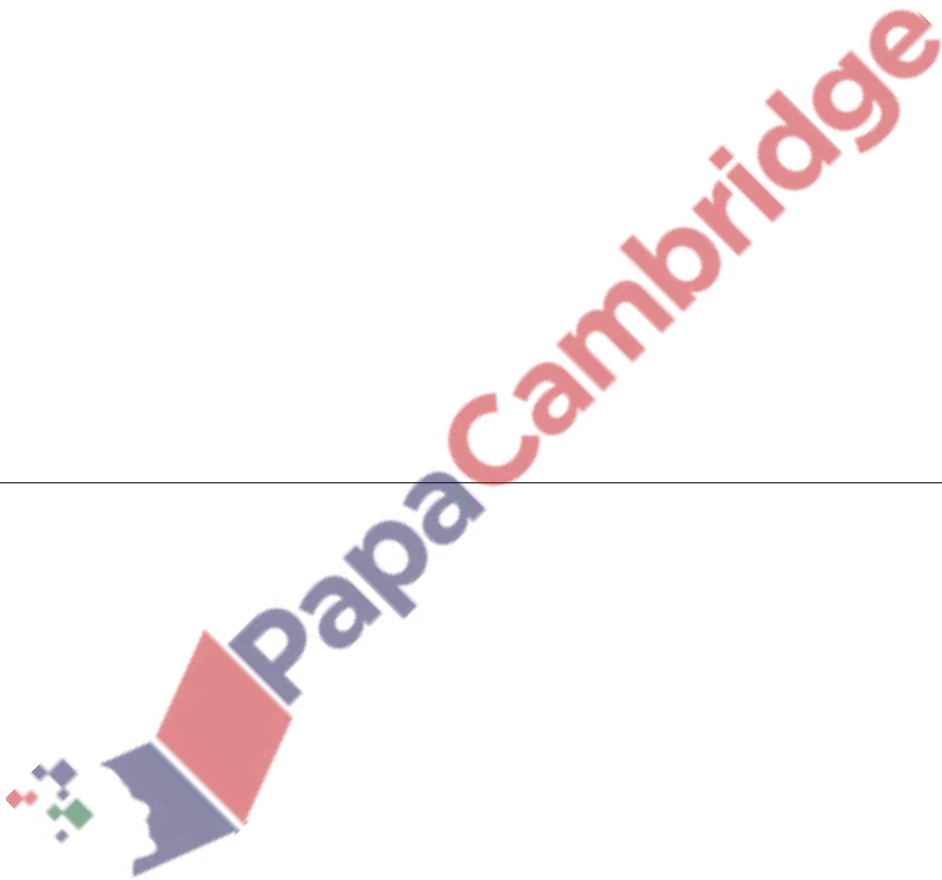
- (i) Sketch this diagram and state fully the geometrical relationship between OB and AC . [4]
- (ii) Find, in the form $x + iy$, where x and y are real, the complex number $\frac{u}{v}$. [3]
- (iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]



346. 9709_w16_qp_31 Q: 9

Throughout this question the use of a calculator is not permitted.

- (a) Solve the equation $(1 + 2i)w^2 + 4w - (1 - 2i) = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]
- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - i| \leq 2$ and $-\frac{1}{4}\pi \leq \arg z \leq \frac{1}{4}\pi$. [5]



347. 9709_w16_qp_33 Q: 7

Throughout this question the use of a calculator is not permitted.

The complex number z is defined by $z = (\sqrt{2}) - (\sqrt{6})i$. The complex conjugate of z is denoted by z^* .

(i) Find the modulus and argument of z . [2]

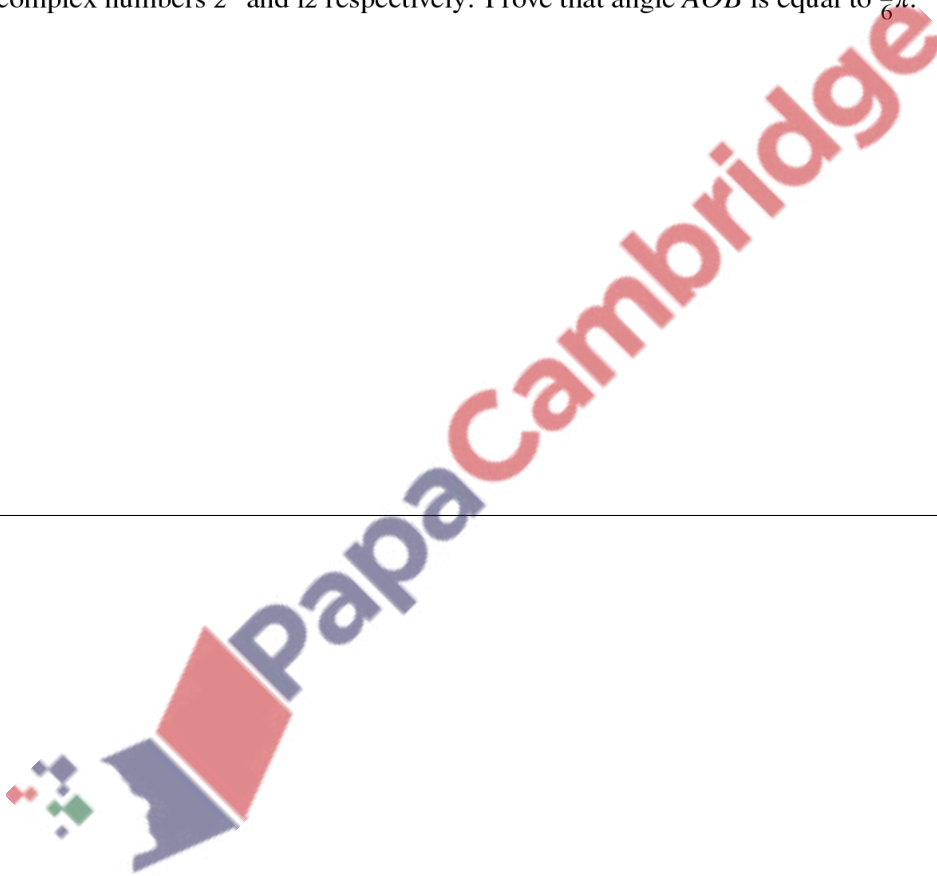
(ii) Express each of the following in the form $x + iy$, where x and y are real and exact:

(a) $z + 2z^*$;

(b) $\frac{z^*}{iz}$.

[4]

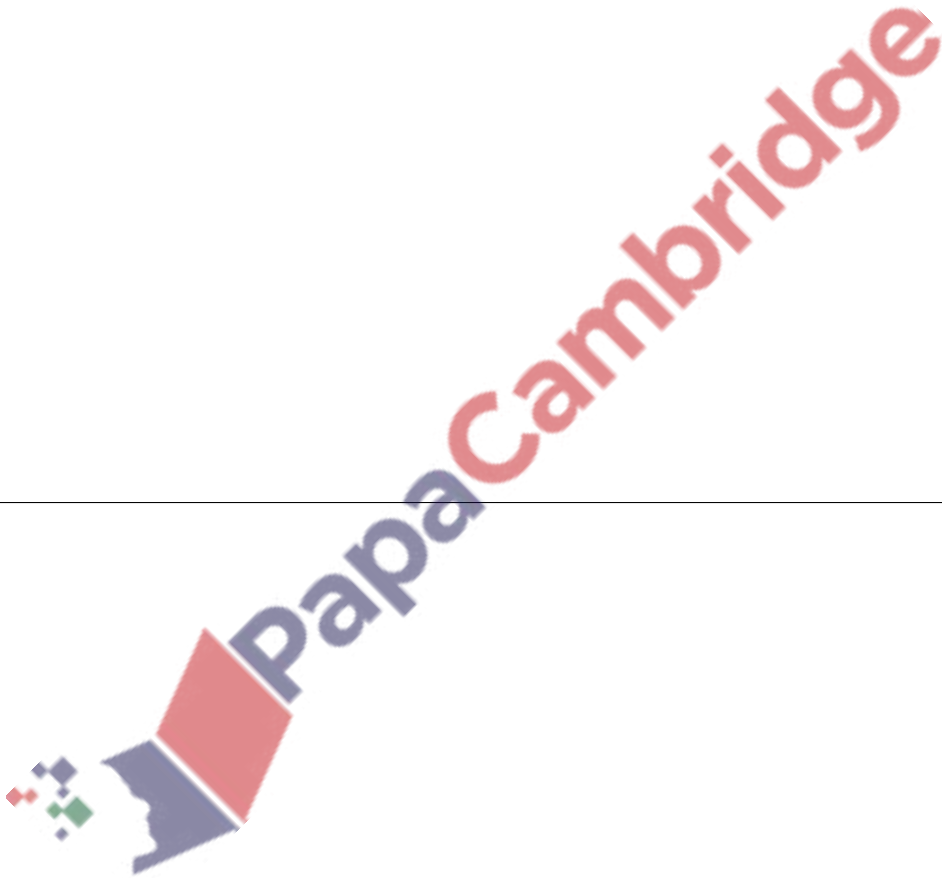
(iii) On a sketch of an Argand diagram with origin O , show the points A and B representing the complex numbers z^* and iz respectively. Prove that angle AOB is equal to $\frac{1}{6}\pi$. [3]



348. 9709_s15_qp_31 Q: 8

The complex number w is defined by $w = \frac{22 + 4i}{(2 - i)^2}$.

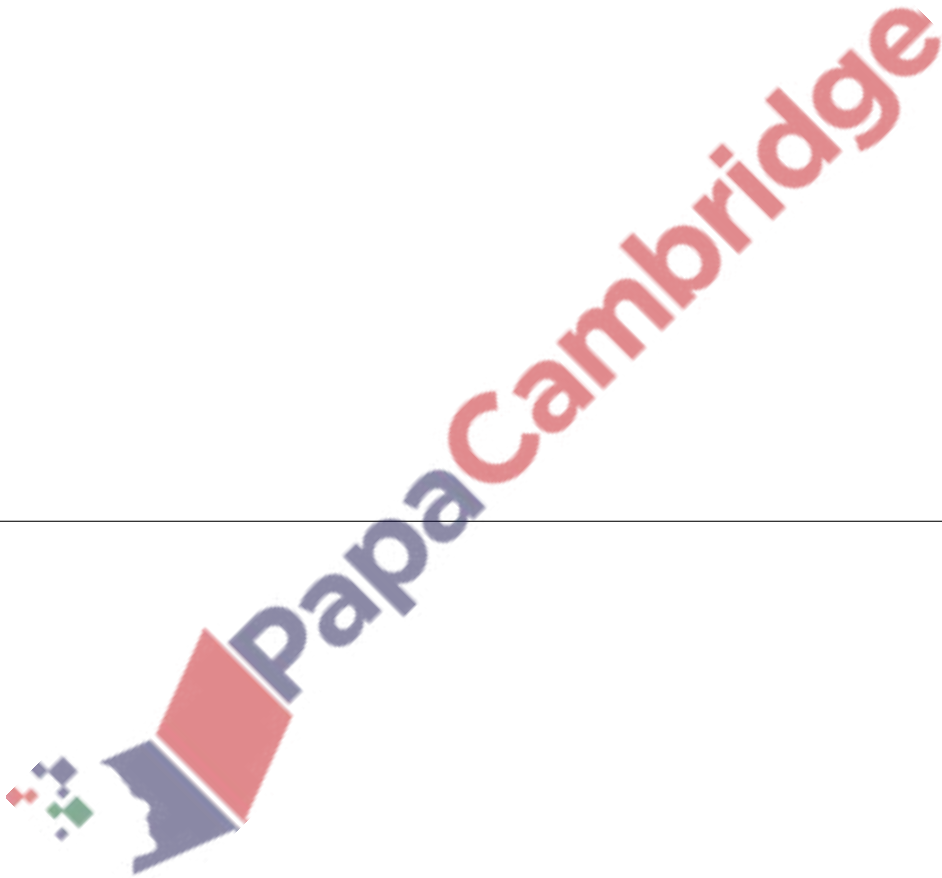
- (i) Without using a calculator, show that $w = 2 + 4i$. [3]
- (ii) It is given that p is a real number such that $\frac{1}{4}\pi \leq \arg(w + p) \leq \frac{3}{4}\pi$. Find the set of possible values of p . [3]
- (iii) The complex conjugate of w is denoted by w^* . The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S , T and the origin. [3]



349. 9709_s15_qp_32 Q: 7

The complex number u is given by $u = -1 + (4\sqrt{3})i$.

- (i) Without using a calculator and showing all your working, find the two square roots of u . Give your answers in the form $a + ib$, where the real numbers a and b are exact. [5]
- (ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]



350. 9709_s15_qp_33 Q: 8

The complex number $1 - i$ is denoted by u .

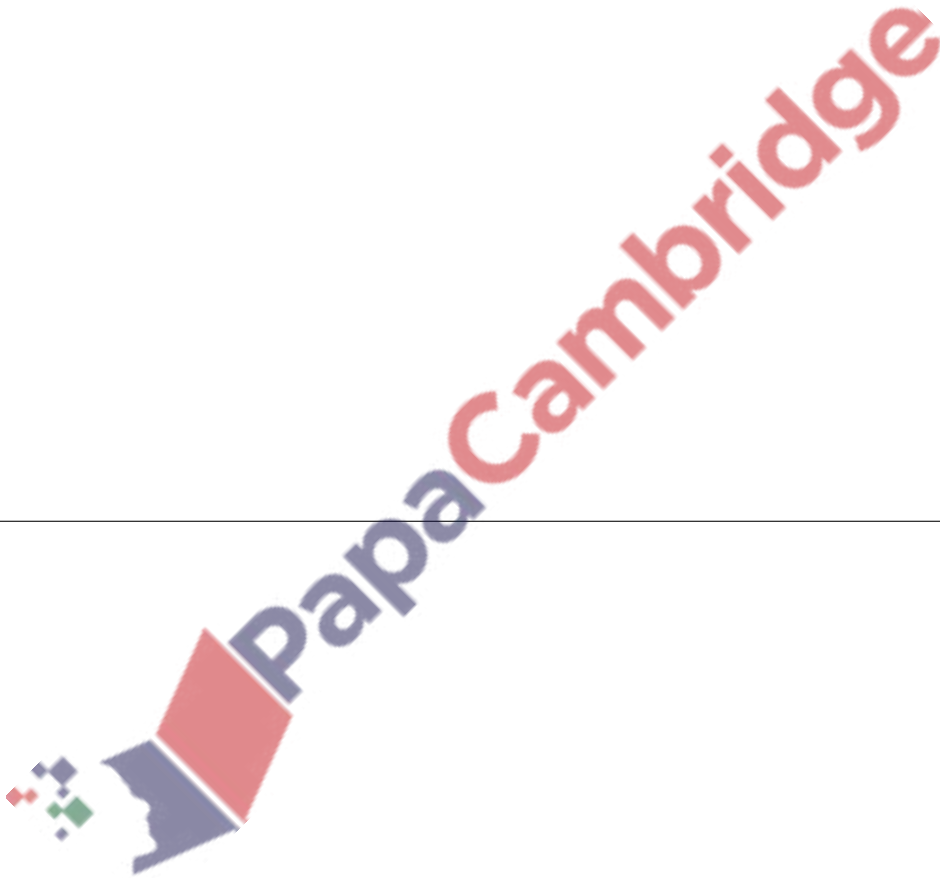
- (i) Showing your working and without using a calculator, express

$$\frac{i}{u}$$

in the form $x + iy$, where x and y are real. [2]

- (ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - i| = 2$. [4]

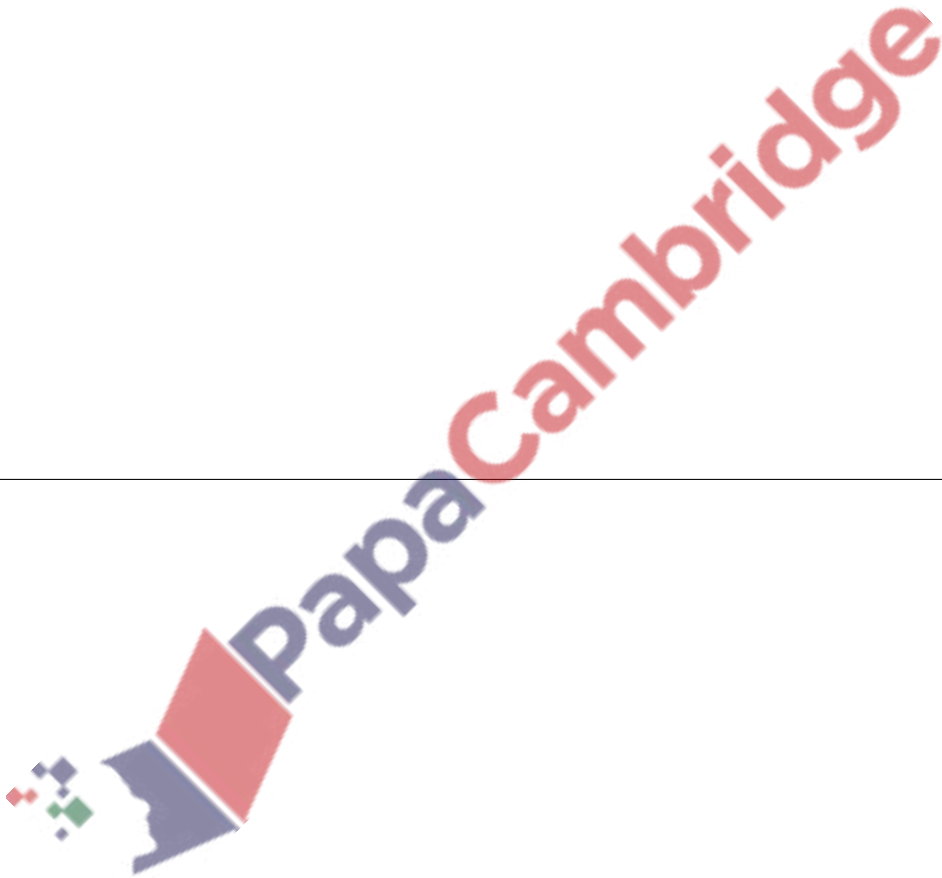
- (iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part (ii). [3]



351. 9709_w15_qp_31 Q: 9

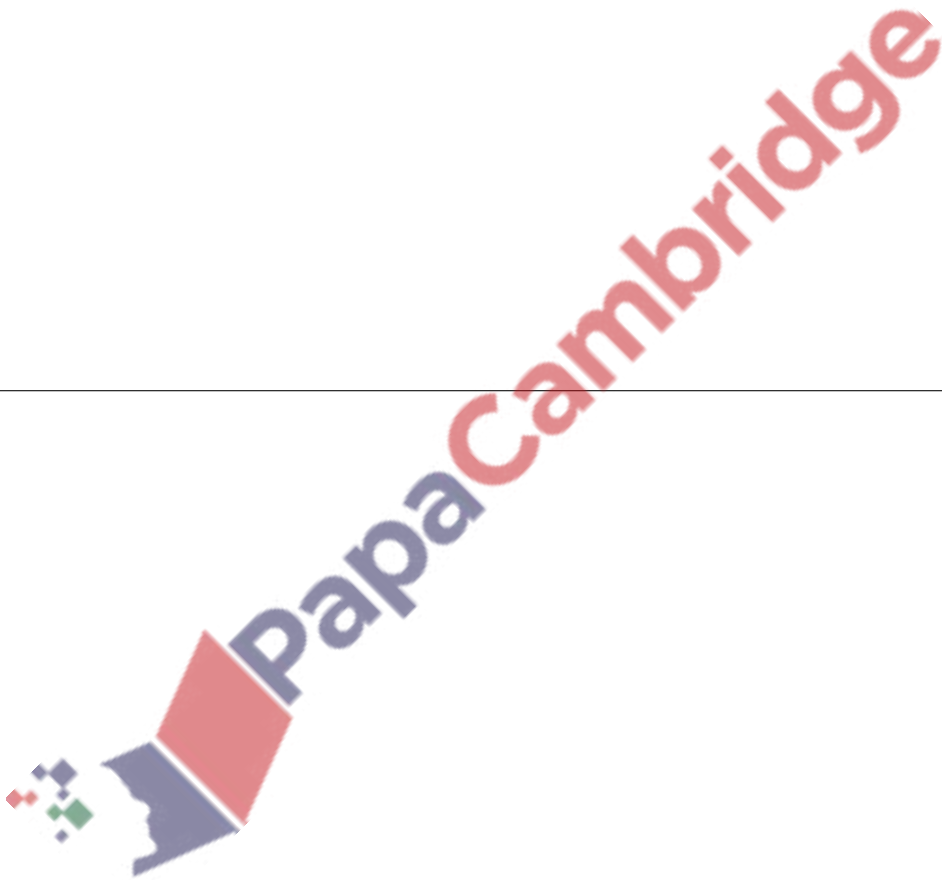
The complex number $3 - i$ is denoted by u . Its complex conjugate is denoted by u^* .


- (i) On an Argand diagram with origin O , show the points A , B and C representing the complex numbers u , u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]
- (ii) Showing your working and without using a calculator, express $\frac{u^*}{u}$ in the form $x + iy$, where x and y are real. [3]
- (iii) By considering the argument of $\frac{u^*}{u}$, prove that
$$\tan^{-1}\left(\frac{3}{4}\right) = 2 \tan^{-1}\left(\frac{1}{3}\right).$$
 [3]



352. 9709_w15_qp_33 Q: 9

- (a) It is given that $(1 + 3i)w = 2 + 4i$. Showing all necessary working, prove that the exact value of $|w^2|$ is 2 and find $\arg(w^2)$ correct to 3 significant figures. [6]
- (b) On a single Argand diagram sketch the loci $|z| = 5$ and $|z - 5| = |z|$. Hence determine the complex numbers represented by points common to both loci, giving each answer in the form $re^{i\theta}$. [4]



A large, semi-transparent watermark of the PapaCambridge logo is oriented diagonally across the page. The logo consists of a stylized 'P' made of colored squares (red, blue, green) followed by the text 'PapaCambridge' in a bold, sans-serif font.